Homework 4

1. **p.258 Ex.11 (a)(b)**
2. By Theorem 2.23 we can see…

1. We let be the matrix in question, and let be the only eigenvalue of the matrix . By Theorem 5.2 we know that the basis to make diagonalisable is , (where is the vectors of which consists of). This gives us that , as forms a basis. Therefore, as , matrix must be
2. **p.259 Ex.14**

By Theorem 4.8 we can see…

1. **p.259 Ex.18(a)(b)**
2. If is invertible, we have that exists, and thus .

We know that , where is the identity matrix

Using the properties of determinants and the fact that , we can simplify that to…

We then have that , we then subtract from both sides of the equation, we get…

So for the scalar value of , then , therefore there is no inverse

1. By part (a) we know that cannot be invertible, then we have that is invertible

(not invertible) (invertible)

We then have…

1. **p.282 Ex.18(a)(b)**
2. According to Theorem 5.1, we know that are the diagonal matrices where the diagonal entries are the eigenvalues of T and U. We know that diagonal matrices are commutative in nature, therefore .

Thus T & U commute

1. We know that there exists an invertible matrix where and such that both and are both diagonal matrices. Since we know that diagonal matrices are commutative (by properties), we get that…

.

, that is and commute.

1. **p.322 Ex.4**

Given that , then , (generally), for

Since is a T-invariant subspace, we then have…

,

Therefore for any , we know that the sum of some elements is in .

is a subspace, we know that is always an element of .

1. **p.322 Ex.6(a)**

We then know that the dimension and, the set is the basis.

1. **p.323 Ex.18(a)(b)**
2. From the equation given in the question we can easily conclude, .

Then by definition we know , and so .

Therefore, is invertible if and only if .

is invertible

1. **p.243 Ex.6.19 [HK]**

characteristic polynomial

Thus, the eigen values of are,

;

The eigenvectors are ,

with

And

1. **p.243 Ex.6.21(1) [HK]**

By lemma 6.10 we know that the characteristic polynomial is…

Therefore, , and the geometric sequences are linearly independent and by Theorem 6.13 the general solution is a linear combination of them…

1. **p.243 Ex.6.27(1) [HK]**

and

the eigenvalues are

The eigenvectors are:

The general solution is…

Where

Hence the final solution is…